Standard Deviation DS Solutions

1. (1) At least one of the integers is positive.

   We can have three cases:
   (i) All three integers are positive. In this case the product will obviously be positive.
   (ii) Two of the integers are positive: {0, 1, 2}. In this case the product will be zero.
   (iii) Only one of the integers is positive: {-1, 0, 1}. In this case the product will be zero.

   Not sufficient.

   (2) The sum of the integers is less than 6. Clearly insufficient, consider {-1, 0, 1} and {-3, -2, -1}.

   (1)+(2) The second statement implies that we cannot have case (i) from (1), since the least sum of three positive consecutive integers is 1+2+3=6. Thus we have either case (ii) or case (iii). Therefore the product of the integers is zero. Sufficient.

   Answer: C.

2. (1) The sum of the terms in set S is 105. Clearly insufficient. For example, consider S={28, 35, 42} and {49, 56}.

   (2) The standard deviation of set S is equal to 3.5. Important property: **if we add or subtract a constant to each term in a set SD will not change**. From this it follows, that:

   Any set with two consecutive multiples of 7 will have the same standard deviation. For example, ..., {0, 7}, {7, 14}, {14, 21}, {21, 28}, ... will have the same standard deviation.
   Any set with three consecutive multiples of 7 will have the same standard deviation. For example, ..., {0, 7, 14}, {7, 14, 21}, {14, 21, 28}, {21, 28, 35}, ... will have the same standard deviation.
   Any set with four consecutive multiples of 7 will have the same standard deviation. For example, ..., {0, 7, 14, 21}, {7, 14, 21, 28}, {14, 21, 28, 35}, {21, 28, 35, 42}, ... will have the same standard deviation.
   ...

   We know the standard deviation of S is 3.5. We CAN get the standard deviations of {0, 7}, {0, 7, 14}, {0, 7, 14, 21}, ... Only one of them will have the standard deviation of 3.5. So, we can get how many terms are there in the set. Sufficient.

   Answer: B.

3. The median of a set with odd (5) number of elements is the middle term, so we have that the heights in ascending order are {a, b, 118, c, d} ($a \leq b \leq 118 \leq c \leq d$)

   (1) The average height of the children in family A is 120cm --> the sum of the heights is 120*5=600 cm:

If the heights are {10, 12, 118, 130, 130}, then 2 children are taller than 128 cm.
If the heights are {118, 118, 118, 122, 124}, then no child is taller than 128 cm.

Not sufficient.

(2) The second highest child in family A is 130cm --> c=130 --> $a \le b \le 118 \le 130 \le d$ --> 2 children, c and d, are taller than 128cm. Sufficient.

Answer: B.

4. To determine the standard deviation of list M we must know which 2 integers were removed.

   (1) The average (arithmetic mean) of the numbers in list M is equal to the average of the numbers in the list shown --> the mean of list M is also 13. Thus the sum of the integers in list M is 13*8=104, which means that the sum of the 2 integers removed is 130-104=26. The 2 integers removed could be: (4, 22), (6, 20), ..., (12, 14). Not sufficient.

   (2) List M does <u>not</u> contain 22. We know only one of the numbers removed. Not sufficient.

   (1)+(2) From (1) we know that the the sum of the 2 integers removed is 26 and from (2) we know that one of the integers removed is 22. Therefore the second integer removed is 26-22=4. List M consists of the following 8 integers: {6, 8, 10, 12, 14, 16, 18, 20}. So, we can determine its standard deviation. Sufficient.

   Answer: C.

5. Standard deviation shows how much variation there is from the mean. A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values.

   We need to know couple of important properties for this question:

   If we add or subtract a constant to each term in a set:
   Mean will increase or decrease by the same constant.
   SD will not change.

   If we increase or decrease each term in a set by the same percent (multiply all terms by the constant):
   Mean will increase or decrease by the same percent.
   SD will increase or decrease by the same percent.

   You can try it yourself:
   SD of a set: {1,1,4} will be the same as that of {5,5,8} as second set is obtained by adding 4 to each term of the first set.

   That's because Standard Deviation shows how much variation there is from the mean. And when adding or subtracting a constant to each term we are shifting the mean of the set by this constant (mean will increase or decrease by the same constant) but the variation from the mean remains the same as all terms are also shifted by the same constant.

Also: **If Range or SD of a list is 0, then the list will contain all identical elements. And vise versa: if a list contains all identical elements then the range and SD of a list is 0. If the list contains 1 element: Range is zero and SD is zero.**

That's because if a list contains all identical elements then there is no variation from the mean, hence SD=0.

BACK TO THE ORIGINAL QUESTION:

What is SD of given set of numbers whose average is 5?

(1) None of the numbers are greater than this Average --> if no number is more than the mean then no number is less than mean, which implies that this list contains all identical elements (or which is the same just one element), so SD=0. Sufficient.

(2) The Standard deviation is **0** when value of each of the given number is increased by 7 --> if we add or subtract a constant to each term in a set SD will not change, so SD=0. Sufficient.

Answer: D.

6. (1) The integer 25 is in list S. The 5th integer in S is 80-25=55. We know list S. Not sufficient.
   (2) The integer 45 is in list T. The 5th integer in T is 80-45=35. We know list T. Not sufficient.

   (1)+(2) We know all terms of each set, thus we can get the standard deviation of each and compare. Sufficient.

   Answer: C.

7. Two very important properties of standard deviation:

   If we add or subtract a constant to each term in a set:
   Mean will increase or decrease by the same constant.
   SD will not change.

   If we increase or decrease each term in a set by the same percent (multiply all terms by the constant):
   Mean will increase or decrease by the same percent.
   SD will increase or decrease by the same percent.

   You can try it yourself:
   SD of a set: {1,1,4} will be the same as that of {5,5,8} as second set is obtained by adding 4 to each term of the first set.

   That's because Standard Deviation shows how much variation there is from the mean. And when adding or subtracting a constant to each term we are shifting the mean of the set by this constant (mean will increase or decrease by the same constant) but the variation from the mean remains the same as all terms are also shifted by the same constant.

Back to the original question:

There is a set of consecutive even integers. What is the standard deviation of the set?

(1) There are 39 elements in the set --> SD of a set of ANY 39 consecutive even integers will be the same, as any set of 39 consecutive even integers can be obtained by adding constant to another set of 39 consecutive integers. For example: set of 39 consecutive integers {4, 6, 8, ..., 80} can be obtained by adding 4 to each term of another set of 39 consecutive integers: {0, 2, 4, ..., 76}. So we can calculate SD of {0, 2, 4, ..., 76} and we'll know that no matter what our set actually is, its SD will be the same. Sufficient.

(2) The mean of the set is 382 --> knowing mean gives us nothing, we must know the number of terms in the set, as SD of {380, 382, 384} is different from SD of {378, 380, 382, 384, 386}. Not sufficient.

Answer: A.

8. (1) say y = {3,3,3}, z = {1,1,1}
Same SD, but Avg(y) > Avg(z)
say y = {1,3,5}, z = {1,1,1}
Different SD, but Avg(y) > Avg(z)
INSUFFICIENT

(2) Using the same case above, INSUFFICIENT . Answer is E.

9. We have the set with 10 terms: {0, x, x^2, x^3, ..., x^9}.

Note that if $x=odd$ then the set will contain one even (0) and 9 odd terms (as if $x=odd$, then $x^2=odd$, $x^3=odd$, ..., $x^9=odd$) and if $x=even$ then the set will contain all even terms (as if $x=even$, then $x^2=even$, $x^3=even$, ..., $x^9=even$).

Also note that: standard deviation is always more than or equal to zero: $SD\geq 0$. **SD is 0 only when the list contains all identical elements (or which is same only 1 element).**

(1) The mean of the set is even --> mean=sum/10=even --> sum=10*even=even --> 0+x+x^2+x^3+...+x^9=even --> x+x^2+x^3+...+x^9=even --> x=even (if x=odd then the sum of 9 odd numbers would be odd) --> all 10 terms in the set are even. Sufficient.

(2) The standard deviation of the set is 0 --> all 10 terms are identical --> as the first term is 0, then all other terms must equal to zero --> all 10 terms in the set are even. Sufficient.

Answer: D.

10. 1) If T is {5,10,15} then standard deviation is positive. If T is {5,5,5} then standard deviation is 0. Insufficient

    2)T contains only one number. Hence standard deviation can only be 0. Sufficient.

    Answer is B.

11. (1) Set A contains five consecutive integers. A={1, 2, 3, 4, 5} and B={1, 2, 4, 5}, then SD(A)<SD(B) (since A is obtained by adding the element equal to the mean to set B) but if A={1, 2, 3, 4, 5} and B={1, 2, 3, 4}, then SD(A)>SD(B) (since B is less widespread than A). Not sufficient.

    (2) The average (arithmetic mean) of set A is equal to the average (arithmetic mean) of set B. If we add to a set new element, which is equal to the mean of the original set, then the standard deviation decreases (well if the SD was not 0). We know that A is obtained by adding one element to set B. Since doing so the mean of A does not change, then we added the element which was equal to the mean, thus we decreased the standard deviation --> SD(A)<SD(B). Sufficient.

    Answer: B.

12. (1) Set Q contains 21 terms --> SD of ALL sets with 21 consecutive integers will be the same, as any set of 21 consecutive integers can be obtained by adding constant to another set of 21 consecutive integers. For example: set of 21 consecutive integers {4, 5, 6, ..., 24} can be obtained by adding 4 to each term of another set of 21 consecutive integers: {0, 1, 2, ..., 20}. So we can calculate SD of {0, 1, 2, ..., 20} and we'll know that no matter what our set actually is, its SD will be the same. Sufficient.

    (2) The median of set Q is 20. Clearly insufficient.

    Answer: A.

13. 1) The arithmetic mean and the median are both 2 --> if the set is {2, 2, 2, 2} then the SD=0 but if the set is {0, 2, 2, 4} then the SD>0. Not sufficient.

    Notice that from this statement we know that since the mean of 4 integers is 2 then the sum of those integers is 2*4=8

    (2) The mode is 2, and the range is also 2--> the standard deviation of {0, 2, 2, 2} is different from the standard deviation of {1, 2, 2, 3}. Not sufficient.

    (1)+(2) We have that mean=median=mode=range=2. Now, median=mode=2 means that the two middle terms must be 2, so our set is {a, 2, 2, b}. Next, since range=2 --> b-a=2 and since mean=2 --> a+2+2+b=8 --> a+b=4. We can solve for a and b, so we'll have all terms in the set, hence we can calculate the standard deviation. Sufficient.

    Answer: C.

14. We are given a set with 6 terms and the standard deviation of 4.5. Next, we are told that each term of this set was increased and are asked to find the new standard deviation of this changed set.

    (1) In the last two years, the heights of Jeremy□s six children have increased a total of 17 inches. Not sufficient, since many different scenarios of increase of each data point are possible.

    (2) In the last two years, each child□s height has increased by 5 percent. Important property: **if we increase or decrease each term in a set by the same percent (multiply all terms by the constant): SD will increase or decrease by the same percent.** So, since each of 6 terms has increased by 5% then the standard deviation of 4.5 would also increase by 5% and become 4.5*1.05. Sufficient.

    Answer: B.

15. **Set T consists of a certain number of even integers divisible by 3. Is standard deviation of T positive?**

    The standard deviation of a set shows how much variation there is from the mean, how widespread a given set is. So, a low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data are spread out over a large range of values. So, basically we can say that it in a sense measures the distance and the distance cannot be negative, which means that **the standard deviation of any set is greater than or equal to zero**: $SD \geq 0$.

    Next, the standard deviation of a set is zero if and only the set consists of identical numbers (or which is the same if the set consists of only one number).

    (1) All elements of set T are positive --> set T can be {6, 6} so with the standard deviation equal to zero or {6, 12} so with the standard deviation more than zero. Not sufficient.

    (2) The range of set T is 0 --> in order the range to be zero set T should have all identical elements, which means that the standard deviation of the set is zero. Sufficient.

    Answer: B.

16. (1) The Average (Arithmetic Mean) number of orders that Website W received each per day for the past 5 days is equal to the greatest of the number of orders that Website W received daily for the past 5 days.

    Since mean=greatest, then the number of orders each day was the same. This implies that the standard deviation is 0 (the standard deviation of a set is zero if and only the set consists of identical numbers). Sufficient.

(2) The range of the number of orders that Website W received daily for the past 5 days is Zero.

Basically the same here: since the range is 0 (greatest-smallest=0 --> greatest=smallest), then the number of orders each day was the same. Sufficient.

Answer: D.

17. Symmetric about the mean means that the shape of the distribution on the right and left side of the curve are mirror-images of each other.

    (1) 68% of the distribution lies in the interval from m-d to m+d, inclusive --> 100%-68%=32% is less than m-d and more than m+d. As distribution is symmetric about the mean then exactly half of 32%, or 16%, would be more than m+d. Sufficient.

    (2) 16% of the distribution is less than m-d --> again, as distribution is symmetric about the mean then exactly 16%, will be more than m+d. Sufficient.

    Answer: D.

18. The standard deviation of a set is always more than or equal to zero ($SD \geq 0$). The standard deviation is 0 only when a set contains all identical elements (or which is same only 1 element).

    (1) The average (arithmetic mean) number of eggs for the 10 nests was 4. Knowing the average does not help to get the standard deviation. Not sufficient.

    (2) Each of the 10 nests contained the same number of eggs. So, the set contains all identical elements, which means that the standard deviation is zero. Sufficient.

    Answer: B.

19. **CALCULATING STANDARD DEVIATION OF A SET {x1, x2, ... xn}:**
    1. Find the mean, m, of the values.
    2. For each value x_i calculate its deviation ($m-x_i$) from the mean.
    3. Calculate the squares of these deviations.
    4. Find the mean of the squared deviations. This quantity is the variance.
    5. Take the square root of the variance. The quantity is th SD.

    Expressed by formula: $standard\ deviation = \sqrt{variance} = \sqrt{\frac{\sum(m-x_i)^2}{N}}$.

    (1) The variance for the set of measurements is 4. The variance is just the square of the standard deviation, so if $variance = 4$ then $SD = \sqrt{4} = 2$. Sufficient.

    (2) For each measurement, the difference between the mean and that measurement is 2. So,

we have that $m - x_i = 2$, since N=20 then we know everything to calculate the standard deviation. Sufficient.

Answer: D.

20. (1) For each tank, 30% of the volume of water that was in the tank at the beginning of the experiment was removed during the experiment.
(2) The average (arithmetic mean) volume of water in the tanks at the end of the experiment was 63 gallons.

You should know that:
If we add or subtract a constant to each term in a set:
Mean will increase or decrease by the same constant.
SD will not change.

If we increase or decrease each term in a set by the same percent (multiply all terms by the constant):
Mean will increase or decrease by the same percent.
SD will increase or decrease by the same percent.

You can check it yourself:
SD of a set: {1,1,4} will be the same as that of {5,5,8} as second set is obtained by adding 4 to each term of the first set.

That's because Standard Deviation shows how much variation there is from the mean. And when adding or subtracting a constant to each term we are shifting the mean of the set by this constant (mean will increase or decrease by the same constant) but the variation from the mean remains the same as all terms are also shifted by the same constant.

So according to this rules statement (1) is sufficient to get new SD, it'll be 30% less than the old SD so 7. As for statement (2) it's clearly insufficient as knowing mean gives us no help in getting new SD.

Answer: A.

21. (1) Z - X = 10. No info about y. Not sufficient.
(2) Z - Y = 5. . No info about x. Not sufficient.

(1)+(2) From above x = z - 10 and y = z - 5, so the set in ascending order is {z-10, z-5, z}. Now, **if we add or subtract a constant to each term in a set the standard deviation will not change**. Adding 20-z to each term in the set we get {10, 15, 20}. So, the standard deviation of {z-10, z-5, z} is equal to that of {10, 15, 20}. Sufficient.

Answer: C.

22. 1. Standard deviation = 3 since mean = 10... 3 units below and 3 units above. can calculate range of 2 standard deviations above and below mean by: 10-6 = 4 and 10+6 = 16... range of 2 standard deviations above and below mean = (4, 16)

    2. Gives no information on standard deviation. In sufficient.

    Answer is A.

23. A store received 7 crates of oranges. What was the standard deviation of the numbers of oranges in the 7 crates?

    (1) For the 7 crates of oranges, the median of the numbers of oranges was equal to the greatest of the numbers of oranges.
    (2) For the 7 crates of oranges, the range of the numbers of oranges was 0.

24. We need to find whether SD lower than what it was in the beginning of the year

    St 1 says average volume of water had decreased by 20%. So if Total volume of water for 10 reservoirs was 100lts then at the end of the year Volume of 10 reservoirs was 80 lts.

    Case 1 :Now Imagine if 9 reservoirs see an increase of 2 lts each and 1 reservoirs sees a decrease of 38 Lts then volume of water is still 80 lts (Average volume of water is 8 lts) but SD will be different (lower or more is not important)
    Case 2:Also another case where each reservoir has volume of 10 lts each and each sees uniform decrease of 20% ie. at the end of the year each reservoir has 8 lts and Total volume is 80 lts and average volume is 8 ltr--------> In this case we can safely say SD is lower but not in Case 1

    So ST 1 is not sufficient

    Consider St 2 points out that each reservoir saw percent decrease and is pointing to Case 2 of St 1 and hence it is sufficient. It is in line with Pt 7. Answer is B.